Empirical Evidence on Competition and Revenue in an All-Pay Contest[†]

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Abstract: The total revenue from an "all-pay contest" is the sum of expenditures from all individual players, so it is important to ask whether it increases with the number of actual players – which is our definition of competition. This is the first paper to use field data to study this question empirically. Using novel instrumental variables, we document strong empirical evidence that the revenue of a penny auction – which is a form of all-pay contest that recently emerged on the Internet – increases with the number of bidders. Our findings cast doubt on the standard model of all-pay contests that presumes that all bidders are fully informed.

Keywords: all-pay contest, penny auction, competition, number of bidders, revenue

JEL Classification: D03, D44, L81

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1. Introduction

Rent-seeking contests, research-and-development races, and political campaigns are all examples of "all-pay contests" (Che and Gale 1998; Siegel 2009): games in which each agent has to incur expenditures to compete for rewards, whether or not that agent wins the rewards. Because the aggregate expenditure in an all-pay contest—or the revenue, from the auctioneer's perspective—is the sum of expenditures from all individual players, a critical question for understanding such all-pay contests is how the number of participants – which is our measure of competition – affects the auctioneer's revenue.

Standard Nash equilibrium analysis of an all-pay auction with a known common value predicts that the expected revenue of an all-pay auction is the value of the prize.[1] However, a large number of experimental studies of all-pay contests find that subjects' total expenditure substantially exceeds the value of the prize. This inconsistency between theory and experimental evidence naturally raises the question: Does greater competition – i.e., a larger number of bidder-participants – increase revenue in an all-pay contest in the field? This paper provides the first empirical study of the effect of increased competition on revenue in an all-pay contest in the field.

We study penny auctions: an all-pay selling mechanism that emerged recently on the Internet. In a penny auction, the price of the auctioned product is initially zero; and, whenever a new bid is placed, the price increases by a small increment (typically 1 cent, hence the name of penny auction). Each bid – an offer to buy the product at the current auction price – costs a nonrefundable fee (e.g., $0.75). The winner is the last bidder: the one whose bid is not followed

[1] In an all-pay auction, all bidders must pay whether they win or lose, and the winner is the bidder who bids the most. Because each player in such auctions has the incentive to outbid her competitors by only a small amount, pure strategies typically have no equilibrium. In mixed-strategy Nash equilibria, players are just indifferent as to whether to participate, so the total revenue equals the common value of the prize (Baye et al. 1996; Bulow and Klemperer 1999).

by another bid before a set time interval (e.g., 30 seconds) expires. The timer is reset whenever a new bid is placed. Penny auctions are an all-pay contest but not a standard all-pay auction.[2]

A simple rational model of penny auctions also predicts that the total revenue of a penny auction does not depend on the number of participants. However, we find strong empirical evidence that the total revenue of penny auctions increases with the number of actual bidders (our measure of competition).

We use an instrumental variable (IV) approach to overcome a critical econometric issue: The number of actual bidders in an auction is endogenous. Our novel IVs are based on the hour of the day at which an auction started. The hour at which an auction started is correlated with the actual number of bidders in an online penny auction for two reasons. First, the active number of Internet users increases during the morning hours and decreases during the late evening hours. Second, the firm that operated the penny auction website from which we obtained our data failed to adjust fully the number of auctions to the potential number of active Internet users.

The remainder of the paper proceeds as follows: Section 2 reviews the related literature, and Section 3 describes the auction rules and our data set. Section 4 offers brief theoretical discussions, and Section 5 presents our empirical results. Section 6 concludes.

2. Literature Review

All-pay contests include Tullock (1980) rent-seeking contests, all-pay auctions, tournaments, and wars of attrition. We are not aware of any studies that use field data to study empirically the Tullock rent-seeking model or wars of attrition. An empirical literature has studied tournaments, such as sports games (e.g., Brown 2011) or performance competition within

[2] Win or lose, all bidders in a penny auction must pay, so it is an all-pay game or all-pay contest. The winner of a penny auction is the last bidder, who is often not the one who submitted the largest number of bids, so a penny auction is not an all-pay auction, in which the winner must be the bidder who submits the largest aggregate bid.

organizations (e.g., Casas-Arce and Martínez-Jerez 2009). The tournament literature is concerned mostly with the efforts or performance of individual players, not the effect of competition on total expenditure. Carpenter et al. (2008) conduct field experiments to study which type of sealed-bid formats, including the all-pay auction, raises the most money.

Our paper is related to a large number of experimental studies of all-pay contests. The vast majority of experimental studies of the Tullock rent-seeking contest find that subjects' total expenditure substantially exceeds the value of the prize. See Table 1 of Morgan et al. (2012) for a summary of the literature. Gneezy and Smorodinsky (2006) report that subjects in their all-pay auction experiments overbid substantially, and that the degree of overbidding increases with the number of (inexperienced) players in the auctions.

Our paper is also related to a literature that has studied the effects of increased competition on auctioneers' revenue in standard auctions in which the highest bidder wins (e.g., Brannman et al. 1987, Brannman 1996, Gómez-Lobo and Szymanski 2001, Hong and Shum 2002, Onur et al. 2011, Athey et al. 2011, and Kosmopoulou and Zhou 2014). Papers in the literature address (to various degrees) the concern that the number of bidders may be endogenous. Onur et al. (2011), for example, use the IV method to address the endogeneity concern.

Our paper is also related to a small but growing literature that examines penny auctions (Augenblick 2011; Byers et al. 2010; Hinnosaar 2010; Goodman 2012; Platt et al. 2013). These papers studied Swoopo (the first penny auction website) and find that Swoopo made excessive profits; but none studied the relationship between the number of bidders and Swoopo's auction revenues.

In particular, Byers et al. (2010) theorize that a possible reason for Swoopo's excessive profit is that bidders may underestimate the number of competitors. This theory, based on the idea that bidders may have incorrect beliefs, can explain our findings if bidders underestimate the degree of competition more when the number of bidders is larger, though our findings could also result from other explanations. Caldara (2012) uses lab experiments to study penny auctions; one of his findings is that the average revenue from auctions with five players is larger than that from auctions with three players. Caldara's lab finding thus corroborates our empirical results.

3. Auction Rules and Data

Penny auctions have been described by Richard Thaler (2009) in the *New York Times* as "devilish" and a "diabolically inventive" adaptation of Martin Shubik's (1971) dollar auction. The first such website targeting US consumers appeared in 2006, and it quickly spawned a sizable and controversial industry.[3] To see why penny auctions have been called devilish, consider an example in our data set. A bidder won an iPad auction after placing 70 bids (each costing \$0.75), at which point the auction price was \$64.97. The price increment was 1 cent and thus 6,497 bids were placed by all bidders. The winner paid a total cost of \$117.47 (= 70 × \$0.75 + \$64.97) for the iPad, but the website's revenue was \$4,937.72 (= $6{,}497 \times \$0.75 + \$64.97$)!

Our evidence comes from a major penny auction website – BigDeal.com – which operated from November 19, 2009, to early August 2011. One of the largest penny auction websites, it appeared to be a serious business endeavor: It received \$4.5 million in initial funding from well-known venture capital firms (Stone 2009).

The rules of BigDeal auctions were representative of all penny auctions: Prior to bidding in any auction, each bidder had to buy a pack of bid tokens. Each token costs \$0.75. BigDeal

[3] By November 2010, at least 125 penny auction websites targeting US consumers were being monitored by Compete.com (a web traffic monitoring company). According to its data, in November 2010 the total number of unique monthly visitors to these penny auction websites reached 25.1 percent of that to eBay but has since declined sharply. Unlike eBay, penny auction websites sell products themselves. See Wang and Xu (2013) for more details.

always set the initial auction price for any product at $0.00. A bidder had to give up a single nonrefundable token to place a bid, and each bid raises the auction price by a small increment, which was 1 cent in most auctions and 15 cents in some auctions.

BigDeal typically released an auction with an initial countdown clock that would last for 36 hours. The initial countdown clock continued to run if a bid was placed when more than 30 seconds remained on the clock. However, the countdown clock would always be reset to 30 seconds if a bid was placed when less than 30 seconds remained. A bidder won if her bid was not followed by any other bid before the timer expired. In addition to her accumulated bidding costs, the winner would also pay the auction price to obtain the product.

BigDeal offered losing bidders a buy-it-now (BIN) option for all auctions except for a number of iPad and bid token auctions (in which a pack of bid tokens is auctioned). A bidder who exercises the BIN option stops her own bidding and obtains a product identical to the one that is being auctioned by paying the difference between the posted retail price of the product and her thus-far accumulated bidding costs for that auction. BigDeal posted a retail price for any product to be auctioned.

For example, the posted retail price for an iPad auction with the BIN option in our data set was $899.99. A losing bidder in this auction placed 1,067 bids, so her cost of bids was $800.25 (=1,067 × $0.75). This bidder needed to pay only an additional $99.74 (= $899.99 – $800.25) to exercise the BIN option and obtain an iPad identical to the one being auctioned. With the BIN option, this bidder paid the posted retail price of $899.99 to buy an iPad. Without the BIN option, this bidder would have paid $800.25 for nothing.

3.1. Data

Using self-written scripts, we downloaded from BigDeal.com the general information and the bidding history of all auctions that took place between November 19, 2009 (the first day of the website's operation), through August 6, 2011 (two days before the website was closed). BigDeal displayed the bidding history of all live and past auctions on its website – perhaps to mitigate potential concerns of shill bidding.

In our data set, BigDeal successfully conducted a total of 115,379 auctions: 81,345 regular auctions, and 34,034 beginner auctions. Whereas regular auctions were open to all bidders, beginner auctions accepted bids only from new members of the website. Most beginner auctions are token auctions. A total of 218,387 bidders placed a total of 23,819,374 bids. We observe which bidder placed each of these bids and in which auction.

Following the literature on penny auctions, we approximate the value of a product by the retail price of the same product at Amazon.com in June 2011. This is a reasonable approximation because our results rely on within-product variations. In our data set, 61.7 percent of the nontoken auction products are available at Amazon. The Amazon prices for these products are, on average, 78 percent of the retail prices posted by BigDeal. Thus, we assume that the value of a nontoken product without an Amazon price is 78 percent of the retail price posted by BigDeal. Our empirical results remain essentially the same even if we focus only on products that were available at Amazon. Since the cost of a token is \$0.75, we presume its value is \$0.75.[4]

3.2. Revenue Definition and Computation

We use two measures of auctioneer's revenue from each auction: the total profit, and the profit per dollar of product value. The total profit from an auction is the net revenue: the total

[4] There is one exception: The value of bid tokens bought through the BIN option in token auctions is less than \$0.75 because of a usage restriction that was imposed on such bid tokens. We assume that the value of a token bought through the BIN option is $0.8 \times \$0.75$. See Wang and Xu (2013) for more details.

revenue from the auction minus the total value of the products sold by the auction. The products sold by an auction include the products sold through the BIN option. The profit per dollar of product value is defined as the net revenue divided by the value of the product.

We focus on net revenue instead of absolute revenue for three reasons: First, the absolute revenue of an auction varies with the market value of the product being auctioned. By focusing on net revenue, we can better aggregate results across a variety of products. Second, by focusing directly on net revenue, we can observe whether auction revenue exceeds the value of the prize: a major focus of the all-pay auction literature. Third, the absolute revenue – but not necessarily the net revenue – of an auction increases linearly with the number of products sold through the BIN option.[5]

To see how we compute the auctioneer's profit, consider an auction in which the value of the product is v, the final auction price is p, the price increment is k, the cost of a token is c, and the number of actual bidders is M. We observe the number of bids that are placed by each of the M bidders, $\{b_1, b_2, \ldots, b_M\}$. If the BIN option is not available, then the profit, π, of the auction is

$$\pi = p + (p/k) \times c - v. \tag{1}$$

If the BIN option is available, the calculation is more involved because we have to estimate which losing bidders may exercise the BIN option.

Suppose bidder i has placed b_i bids. To exercise the BIN option, she needs to pay additional cost $r - b_i \times c$: the difference between the posted retail price and her bidding cost. A player exercises the BIN option only if the additional cost needed is smaller than the value of the product: v. Early in the auction, players are unlikely to exercise this option because their

[5] If all players are rational and fully informed, the net revenue of a penny auction should be zero – i.e., the absolute revenue equals the value of the item – regardless of the number of bidders who exercise the BIN option. See Section 4 for more explanations.

number of bids is unlikely to reach the threshold level. Therefore, the number of losing bidders who exercise the BIN option in this auction is $\sum_{i \neq j}^{M} I(r - b_i \times c \leq v)$, where j denotes the winner of the auction, and $I(\cdot)$ is an indicator function. Hence, the auctioneer's profit from this auction is

$$\pi = p + b_j c - v + \sum_{i \neq j}^{M} \left[b_i c \times \left(1 - I(r - b_i \times c \leq v)\right) + (r - v) \times I(r - b_i \times c \leq v) \right]. \quad (2)$$

Table 1. Auction Profit from Regular Auctions by Product Category

Product categories	Number of auctions	Total profit ($)	Total value of products sold ($)	BIN frequency per auction	Profit per dollar of product value ($)
iPads (without BIN)	958	$1,507,701	$635,980		$2.37
Bid tokens (without BIN)	11,524	1,099,530	727,830		1.51
Gift cards	2,728	72,597	258,570	1.91	0.28
Bid tokens (with BIN)	8,430	577,986	2,141,130	4.16	0.27
Apple products	3,893	1,297,667	6,108,139	3.17	0.21
Video games and consoles	10,596	216,059	2,194,415	1.00	0.10
Non-Apple electronics	20,990	666,621	8,422,053	1.31	0.08
TVs	1,546	80,255	3,429,954	1.58	0.02
Toys	102	–630	6,268	0.64	–0.10
Housewares	11,519	–233,843	1,488,959	1.04	–0.16
Movies	2,555	–26,393	99,351	0.18	–0.27
Health, beauty, sunglasses, watches	956	–73,387	215,872	0.42	–0.34
Jewelry	2,528	–361,353	726,181	0.17	–0.50
Handbags	3,020	–270,458	519,829	0.16	–0.52
Total or auction-weighted average	81,345	$4,552,351	$26,974,532	1.32	$0.17

Note: The BIN option was available in all auctions except for some auctions of iPads and bid tokens. Non-Apple electronics include computers, cameras, phones, GPS devices, and related electronics.

Table 1 reports auction profits or losses by product category. The total profit of all regular auctions at BigDeal amounted to $4.55 million, so in the aggregate, overbidding occurred. However, overbidding did not occur in auctions for all categories of products. The "Total profit" column of Table 1 indicates that bid tokens and Apple products (with or without the BIN option), gift cards, and non-Apple electronics were profitable, but product categories such as toys, housewares, and movies were not profitable. The far-right column shows a similar pattern in the profit per dollar of product value.

Table 1 is also indicative of the effect of the BIN option on auction profit. The BIN option significantly reduced the profit per dollar of product value for bid token and Apple product auctions. The BIN frequency column of Table 1 indicates that the BIN option was not frequently exercised. This is because the vast majority of bidders at BigDeal placed a small number of bids in any auction, and it is not optimal for them to exercise the BIN option. The vast majority of BigDeal bidders participated in only a few auctions, placed a small number of bids, and then permanently quit the site within a week or so. Of the BigDeal bidders, 75 percent placed no more than 55 bids in total (across a number of auctions), and 95 percent placed no more than 300 bids.

4. Theoretical Considerations

Penny auctions are a complicated dynamic game, and the existing theoretical analyses of this game are all highly stylized. Therefore, we first consider the implications of general economic principles in the context of this game. We then review the basic equilibrium model of penny auctions that has been proposed by Augenblick (2011) and by Platt et al. (2013), which predicts that auction revenue is independent of the number of bidders. We then discuss possible mechanisms through which auction revenue may increase with the number of bidders.

Following the existing analyses of penny auctions, let us suppose that all players have the same valuation of the auctioned product. Players have the choice of not participating in this game, so the principle of individual rationality implies that any model with fully rational, informed, risk-neutral players should predict that the expected revenue of a penny auction cannot exceed the value of the product that is being auctioned. Otherwise, some players would have a negative expected surplus from playing the game.

Firms have the option of not running penny auctions, so the expected revenue cannot be less than the value of the product, either. Therefore, models of penny auctions in which players are fully rational and risk neutral cannot offer the prediction that auction revenue increases with the number of competitors.[6] The equilibrium model reviewed below is a rational model that is in this vein.

Suppose an item is of value v to all bidders, and the number of bidders in the auction, M, is common knowledge. The initial auction price of the item is $p = 0$. Whenever a bidder places a bid, she pays a cost, c, and the auction price is increased by k. Each period, t, is set to last a fixed number of seconds ex ante, but whenever a bid is placed, the period ends and a new period, $t + 1$, starts. A bidder wins if her bid is not followed by another bid before the period ends. For simplicity, assume that all potential bids within a period are placed simultaneously, and if two or more players decide to bid within a period, one player's bid is randomly accepted and only this bidder incurs a bid cost.[7] Assume that the BIN option is not available.

A bidder who places a bid during period t is betting c that no player will bid during period $t + 1$. Assume that $T = (v - c)/k$ is an integer such that the auction price in period T is $v - c$. The game can then be solved by backward induction. No player bids in any period after T because the auction price plus a bid cost exceeds the value of the product. In period T, a bidder is

[6] Auction revenue may depend on the number of bidders in some special cases. For example, the probability that an auction does not start at all may depend on the number of bidders. The greater is the number of bidders, the more likely is the auction to start. Another example is that there may exist some equilibria in which the revenue is strictly lower than the value of the object and the likelihood of obtaining such equilibria depends on the number of bidders.

[7] Hinnosaar (2010) makes the alternative assumption that simultaneous bidders all incur a bid cost. In practice, tying does not occur because time is continuous.

indifferent between bidding and not bidding because, if she bids, she pays $v - c + c$ to win a product of value v. In any period $t < T$, all bidders are assumed to play mixed strategies, and the indifference condition that characterizes the subgame perfect equilibrium is given by

$$(v - tk)\mu_{t+1} = c, \tag{3}$$

where μ_{t+1} is the probability that no one else will bid during period $t + 1$. In equilibrium, a bidder's expected payoff of placing a bid equals the cost of a bid.

For equation (3) to hold in period t, each of the other $M - 1$ symmetric bidders must bid during period $t + 1$ with probability τ_{t+1}, such that the following equation holds:

$$(1 - \tau_{t+1})^{M-1} = \mu_{t+1}^{*} = c/(v - tk). \tag{4}$$

That is, bidders adjust their probabilities of placing a bid by solving equation (4). If the number of bidders is larger, then each individual bidder' probability of bidding must be smaller. Hence, the number of players in an auction affects an individual player's probability of bidding, τ_{t+1}, yet has no effect on μ_{t+1}^{*}, which characterizes the equilibrium. If at least two bids are submitted, the expected revenue for the auctioneer is v because each bidder's expected gain is zero.

The literature has not been able to characterize the equilibrium of the game if the BIN option is available.[8] However, even with the BIN option, it remains true that bidders have the option of not playing and firms have the option of not auctioning. If players are fully rational, informed, and risk neutral, then the principle of rationality still implies that the expected revenue of a penny auction should equal the value of the auctioned product no matter how many players are in the auction. A rational bidder is expected to bid less aggressively if it is common knowledge that the number of bidders is larger.

[8] With the BIN option, it is difficult to characterize the equilibrium because of a complication that arises when a bidder's total number of bids in an auction reaches the level at which she is better off by exercising the BIN option than by not doing so. At that point, a bidder's marginal cost of an additional bid is zero because an additional bid automatically reduces the additional cost she needs to pay to exercise the BIN option. Therefore, a rational bidder may adopt the strategy of always bidding until her total cost of bids reaches the value of the auctioned product.

This model makes several critical assumptions, including the assumptions that: (a) players' beliefs about the number of bidders are correct; and (b) all bidders are homogeneous and fully rational such that they all bid optimally. If either of these two assumptions is relaxed, auction revenue may increase with the number of bidders.

Nash equilibrium is based on two fundamental assumptions: players act optimally given their beliefs about other players' behavior, and their beliefs are correct. Byers et al. (2010) modify the basic equilibrium model of the penny auction to incorporate the possibility that bidders have incorrect beliefs: They underestimate the number of bidders. Given their (incorrect) beliefs, bidders still act optimally. If the degree of underestimation is larger when the number of bidders is greater, this model would predict that total revenue increases with competition.

Another possible mechanism by which competition could increase total revenue is that players have the correct belief about competitors but do not always act optimally given their correct belief. Anderson et al. (1998) provide a logit equilibrium model of all-pay auctions that takes this approach. In their model, actions are chosen by a logit probabilistic rule in which choices with higher expected payoffs are chosen more often, but not always. If players' bidding behavior is not always optimal,[9] they tend to overbid, so revenue would increase with the number of such boundedly rational bidders.

A third possible explanation is that products sold by penny auctions may have a private value component. In all-pay auctions with independent private values, total revenue may increase with the number of competitors. The best way to see why this is the case is through the revenue equivalence theorem. It is well known that the expected revenue of a standard first-price auction with independent private values is the valuation of the second-highest bidder, which increases

[9] Bidders may suffer from various behavioral biases (e.g., the sunk cost fallacy) that lead them to overbid.

with the number of competitors. This result applies to all-pay auctions with independent private values if revenue equivalence holds.

Our paper takes the first step of empirically establishing that penny auction revenue increases with the number of bidders. We suspect that all three mechanisms mentioned above may be relevant for explaining our empirical findings, but it is beyond the scope of this paper to identify which mechanism is more relevant.

5. Empirical Analysis

5.1. Identification Strategy

To study the effect of competition on the auctioneer's profit, we consider the following simple regression specification:

$$\pi_i = \alpha + \beta M_i + \mu_i + \varepsilon_i, \qquad (5)$$

where π_i is the total profit or the profit per dollar of product value from auction i, M_i is the number of actual bidders in auction i (and our measure of competition), μ_i is the product fixed effect, and ε_i is the error term. By controlling for the product fixed effects, we rely on within-product variations to identify the effect.

Our interest lies in estimating the coefficient β, the marginal effect of one more bidder on the auctioneer's profit. The ordinary least squares (OLS) estimate of this coefficient is biased because the actual number of bidders in a penny auction is clearly an endogenous variable. To obtain unbiased estimates of the coefficient, we consider a set of novel instrumental variables.

Our instruments are based on the idea that the number of potential bidders in a BigDeal auction depends on the hour of the day at which the auction started. (We say that an auction starts when the 30-second timer starts.[10]) For example, most individuals in the United States, BigDeal's customer base, wake up and become active Internet users from, say, 6:00 a.m. through 11:59 a.m. Eastern Standard Time (EST), which is 3:00 a.m. through 8:59 a.m. Pacific Standard Time (PST). We thus expect the number of potential bidders at BigDeal to increase during this morning period. Similarly, we expect the number of potential bidders at BigDeal to decrease during the late evening period, from, say, 10:00 p.m. through 3:59 a.m. EST, which is 7:00 p.m. through 12:59 a.m. PST, as people go to bed.

Our IVs are 23 dummy variables, $hour_1, \ldots, hour_{23}$, where $hour_j$ is 1 if the start time of an auction is the jth hour of the day. For example, $hour_1$ is 1 if the start time of an auction is from 1:00 a.m. through 1:59 a.m. EST. The omitted hour is from 0:00 a.m. through 0:59 a.m. EST. In the rest of this paper, we convert all time, including daylight saving time, into EST.

For our IVs to be highly correlated with the actual number of bidders in individual auctions, it must be the case that BigDeal failed to adjust fully the number of auctions to the total number of potential bidders at its website. A comparison of Figures 1(a) and 1(b), below, suggests that BigDeal scheduled fewer auctions such that the 30-second timer would start during the late-night or early-morning hours when fewer potential consumers were active, but the adjustment was not enough.

Figure 1(a) shows the number of regular auctions that started at each hour from the beginning of BigDeal's operation to October 4, 2010. The number of auctions that were scheduled to start at each hour remained roughly the same from 9 a.m. to midnight EST, with an average of 1,899 auctions per hour. The number of auctions that were scheduled to start at each

[10] Recall that BigDeal typically released an auction with an initial countdown clock that would last for 36 hours, and the 30-second timer did not start until only 30 seconds was left on the initial countdown clock. Since nearly all bids at BigDeal were placed after the 30-second timer started, what affected the auction outcome was the point at which the 30-second timer started, not the initial 36-hour countdown clock.

hour between 1 a.m. and 8 a.m. EST was considerably less, with an average of only 1,355 auctions per hour.

Figure 1(b) shows the same information for the period from October 5, 2010, through the end of our data set. On October 5, 2010, the number of auctions that were scheduled to start during the late-night and early-morning hours (from 4 a.m. through 7 a.m. EST) was dramatically reduced; after that day, few auctions were scheduled to start during the early-morning hours.

Figure 1(a). Number of Regular Auctions Whose 30-Second Timer Started at Each Hour (EST, Nov. 19, 2009, to Oct. 4, 2010)

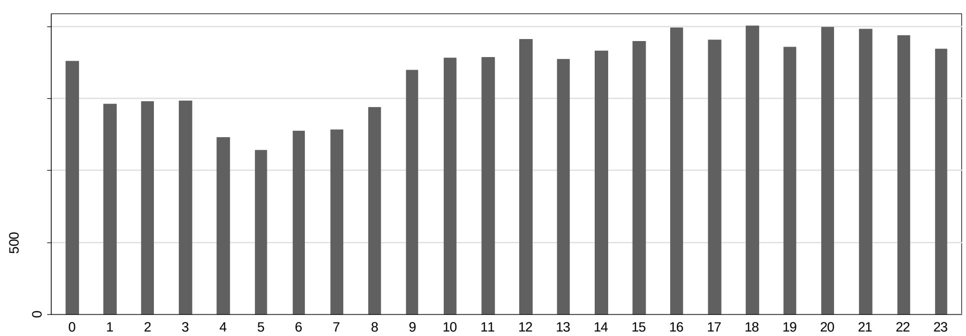
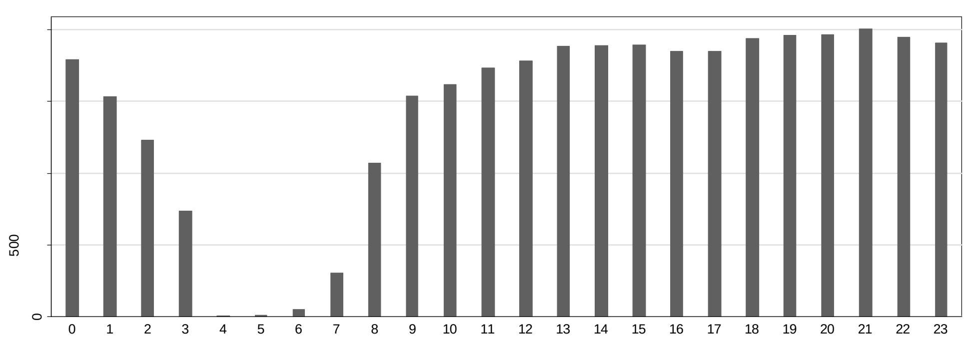


Figure 1(b). Number of Regular Auctions Whose 30-Second Timer Started at Each Hour (EST, Oct. 5, 2010, to Aug. 6, 2011)



Why did BigDeal essentially cease scheduling auctions to start during the four early-morning hours? Our interpretation is that auctions that were started during those hours attracted fewer bidders and generated smaller revenue. That is, BigDeal likely failed to adjust fully the number of auctions to the number of potential bidders, at least for the period on and before October 4, 2010.

We use the following (first-stage) regression to confirm that our IVs are correlated with the number of actual bidders in individual auctions:

$$M_i = c + \delta_1 \cdot Hour_1 + \cdots + \delta_{23} \cdot Hour_{23} + \omega_i + \xi_i, \tag{6}$$

where M_i and $Hour_j$ are as previously defined, ω_i is the product fixed effect, and ξ_i is the error term. We hypothesize that the website failed to adjust fully the number of auctions to the number of potential bidders. Specifically, we hypothesize that the coefficients for the morning hours ($\delta_6, \delta_7, \dots, \delta_{12}$) to exhibit an increasing trend and the coefficients for the late evening hours ($\delta_{22}, \delta_{23}, \dots, \delta_3$) to exhibit a decreasing trend.

Figures 1(a) and 1(b) suggest that the period on and before October 4, 2010, differs from the period afterwards. We shall use the Chow-test to confirm that the two periods are indeed different. For this reason, we estimate equation (6) for two separate sample periods: the period on and before October 4, 2010, and the period afterwards. During the first period, the price increment was 1 cent for most auctions but was 15 cents for a significant number of auctions. During the second period, the price increment was 1 cent for almost all auctions (the price increment was 15 cents for only 31 auctions). Since the size of price increment may affect auction outcome, we examine these two types of auctions separately.

Figures 2(a), 2(b), and 2(b) present the estimated coefficients for the 23 dummy variables as well as the 95 percent confidence intervals. For Figure 2(a), the price increment is 1 cent and the sample period is the first period. For Figure 2(b), the price increment is 1 cent and the sample period is the second period. For Figure 2(c), the price increment is 15 cents and the sample period is the first period. Because of an insufficient number of observations, we do not estimate equation (6) for auctions that had the price increment of 15 cents and that started on and after October 5, 2010.

All three figures confirm our hypothesis: The number of actual bidders per auction exhibits an increasing trend during the morning period and a decreasing trend during the late evening hours. In addition, the number of actual bidders per auction is significantly smaller during the late-night and early-morning hours than during the rest of the day. These results indicate that the website failed to adjust fully the number of auctions to the potential number of bidders, even after October 5, 2010.

Figure 2(a). Coefficients for the Hourly Dummy Variables
(1 Cent Price Increment; EST, Nov. 19, 2009, to Oct. 4, 2010)

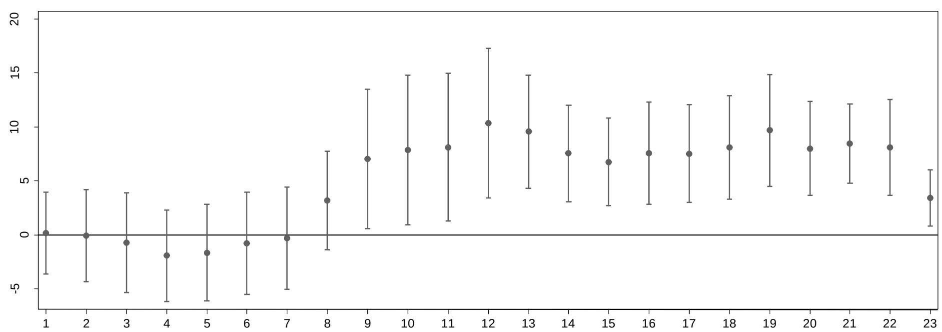


Figure 2(b). Coefficients of 23 Hourly Dummy Variables
(1 Cent Price Increment; EST, Oct. 5, 2010, to Aug. 6, 2011)

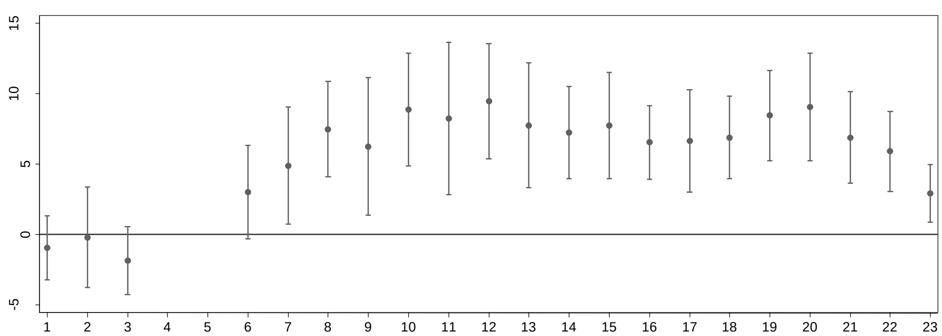
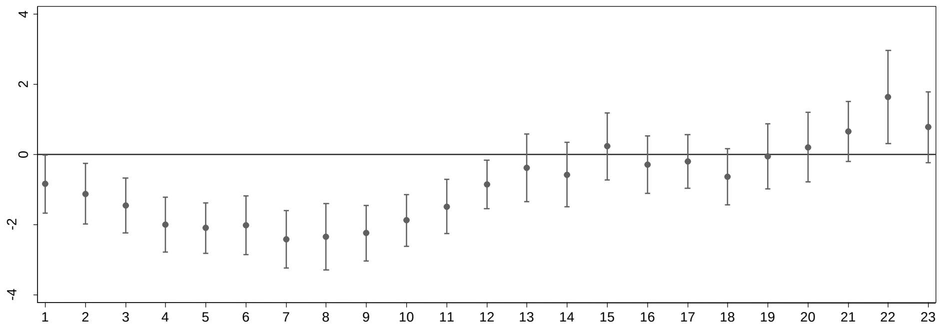


Figure 2(c). Coefficients of 23 Hourly Dummy Variables
(15 Cents Price Increment; EST, Nov. 19, 2009, to Oct. 4, 2010)



To test whether the sample periods before and after October 4, 2010, differ from each other, we define a dummy variable that is 1 for the period on and before October 4, 2010, and interact this dummy variable with the 23 hourly dummies in equation (6). We then add the 23 interaction terms into equation (6), estimate the new equation with all auctions with the price increment of 1 cent, and test whether the 23 interaction terms are jointly significant. The *F* statistic is 5.72, and the *p*-value is 0.000, rejecting the null that the two periods are the same.

We believe that our IVs are uncorrelated with the error term in equation (5), which is the second condition for our IV approach to be valid. Like other researchers, we cannot test formally whether this second condition holds because the error term is unobservable.

The concern that our IVs might be correlated with bidder composition is unwarranted, because the way in which bidder composition changes is different from the way the number of bidders changes. For example, from the early morning hour of 6:00 a.m. to 6:59 a.m. EST to the late morning hour of 11:00 a.m. to 12: 00 noon EST, the number of potential bidders increases, and bidder composition changes from mostly individuals living in the East Coast to individuals living in all areas of the United States. However, from the late evening hour of 10:00 p.m. to 10:59 p.m. EST to the late-night hour of 3:00 a.m. to 3: 59 a.m. EST, the number of potential bidders decreases, but bidder composition changes from individuals living in all areas of the United States to mostly individuals living in the West Coast.

5.2. Results

Panels A and B of Table 2 display the summary statistics of the main variables for regular auctions whose price increment is 1 cent and 15 cents, respectively. As in the previous section, we present the results for two separate sample periods: the period from November 19, 2009 through October 4, 2010, and the period from October 5, 2010, through August 6, 2011. We do not consider the auctions that had the price increment of 15 cents and started during the second period because of the very small number of observations. The samples we consider have a large number of auctions and exhibit large variations in the number of bidders, the profit of an auction, and the profit of per dollar of product value in an auction.

Table 3 reports the regression results for the sample of auctions whose price increment is 1 cent. The sample period for Panel A is from November 19, 2009, to October 4, 2010; and the sample period for Panel B is from October 5, 2010, to August 6, 2011.

Consider first the Panel A results. When the dependent variable is the profit of an auction, the IV estimate of the marginal effect of one more bidder is $7.66, which is much smaller than the OLS estimate of $14.72. When the dependent variable is the profit per dollar of product value, the IV estimate of the marginal effect is $0.061, which is similar to the OLS estimate of $0.046. In Panel B, the same patterns hold: If the dependent variable is the profit of an auction, the IV estimate is much smaller than the OLS estimate; but if the dependent variable is the profit per dollar of product value, the IV estimate is similar to the OLS estimate. The IV and OLS estimates of the marginal effects in Panel B are slightly smaller than those in Panel A if the dependent variable is the profit of an auction; but the Panel A estimates are significantly smaller than the Panel A estimates if the dependent variable is the profit per dollar of product value.

Table 4 reports the results for the sample of auctions whose price increment is 15 cents. We only report the results for the sample period of November 19, 2009, through October 4, 2010, because the number of observations for the second period is too small. When the dependent variable is the profit of an auction, the IV estimate is $9.60, which is larger than the IV estimate in Panel A of Table 3. When the dependent variable is the profit per dollar value of product, the IV estimate is $0.055, which is similar to the IV estimate in Panel A of Table 3.

Table 2. Summary Statistics of Main Variables for Regular Auctions

Variables	Auctions from Nov. 19, 2009, to Oct. 4, 2010				Auctions from Oct. 5, 2010, to Aug. 6, 2011			
	Mean	Std. Dev.	Min	Max	Mean	Std. Dev.	Min	Max
Panel A. Price increment is 1 cent								
Number of bidders in auction	22.62	33.10	1	728	26.51	47.36	1	1,020
Profit of auction (dollars)	76.52	480.48	-4837.63	13,976.56	83.99	612.83	-26,040.09	15,718.52
Profit per dollar of product value	0.56	2.23	-0.99	24.17	-0.03	1.13	-0.999	22.49
Number of products	673				1,098			
Number of auctions	23,761				30,688			
Panel B. Price increment is 15 cents								
Number of bidders in auction	13.17	12.52	1	152	29.55	29.90	3	108
Profit of auction (dollars)	28.05	235.84	-2,289.17	3,027.66	-179.90	663.44	-1,554.59	2,208.36
Profit per dollar of product value	-0.04	0.83	-0.99	6.14	-0.41	0.50	-0.997	0.67
Number of products	465				27			
Number of auctions	14,932				31			

Table 3. Estimated Effects of Competition on Revenue (Price Increment Is 1 Cent)

	Dependent variable			
	Profit of auction		Profit per dollar of product value	
Variables	OLS	IV	OLS	IV
Panel A: Auctions from Nov. 19, 2009, to Oct. 4, 2010				
Number of actual bidders	14.720***	7.662***	0.046***	0.061***
	(1.292)	(1.757)	(0.014)	(0.015)
Constant	-256.512***	-99.640**	-0.494	-0.820**
	(29.242)	(41.800)	(0.315)	(0.347)
Observations	23,761	22,430	23,761	22,430
Number of products	673	671	673	671
R-squared	0.677	0.521	0.292	0.257
Panel B: Auctions from Oct. 5, 2010, to Aug. 6, 2011				
Number of actual bidders	12.412***	7.110***	0.012***	0.009***
	(0.944)	(2.483)	(0.003)	(0.003)
Constant	-245.072***	-109.551***	-0.341***	-0.214***
	(25.028)	(71.149)	(0.086)	(0.083)
Observations	30,688	28,109	30,688	28,109
Number of products	1,098	1,086	1,098	1,086
R-squared	0.669	0.546	0.155	0.146

Notes: The sample covers regular auctions whose price increment is 1 cent. Product fixed effects are included in all regressions. Reported constants are average product fixed effects. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, and * p<0.1.

Table 4. Estimated Effects of Competition on Revenue (Price Increment Is 15 Cents)

Variables	Dependent variable			
	Profit of auction		Profit per dollar of product value	
	OLS	IV	OLS	IV
Number of actual bidders	19.115***	9.600***	0.050***	0.055***
	(1.821)	(2.230)	(0.008)	(0.008)
Constant	-223.629***	-101.472***	-0.693***	-0.737***
	(23.978)	(31.285)	(0.110)	(0.110)
Observations	14,932	13,792	14,932	13,792
Number of products	465	462	465	462
R-squared	0.395	0.297	0.255	0.239

Notes: The sample covers regular auctions whose price increment is 15 cents and the period November 19, 2009, through October 4, 2010. Product fixed effects are included in all regressions. Reported constants are average product fixed effects. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, and * p<0.1.

5.3. Robustness Checks

In this section, we show that our empirical results in the previous section are robust. We first modify the instrumental variables to be of half-hour and quarter-hour intervals. When each instrument covers a half-hour interval, we have 47 half-hour dummy variables. When each instrument covers a quarter-hour interval, we have 95 quarter-hour dummy variables. We then restrict the sample of auctions to those products whose prices were available at Amazon. We also ignore the BIN option when calculating auction revenue.

Table 5 presents the new IV estimates of the marginal effect of one more bidder for auctions whose price increment is 1 cent, using the half-hour and quarter-hour dummies. The IV estimates of the marginal effect on the profit of an auction range from $7.716 to $8.644, which are very similar to the previous IV estimates in Table 3. The new IV estimates of the marginal effect on the profit per dollar of product value range from $0.059 to $0.061 for the first sample period, and range from $0.008 to $0.009 for the second sample period. These estimates are also very similar to the IV estimates in Table 3.

Table 5. Robustness Checks (1-Cent Increment): Alternative IVs

	Instrumental variable: half-hour dummy variables			
	Nov. 19, 2009, to Oct. 4, 2010		Oct. 5, 2010, to Aug. 6, 2011	
	Profit of auction	Profit per dollar of product value	Profit of auction	Profit per dollar of product value
Number of actual bidders	8.133***	0.061***	7.716***	0.009***
	(1.934)	(0.014)	(2.438)	(0.003)
Constant	-110.835**	-0.801**	-126.905*	-0.215***
	(46.003)	(0.340)	(69.871)	(0.079)
Observations	22,430	22,430	28,109	28,109
Number of products	671	671	1,086	1,086
R-squared	0.541	0.261	0.573	0.146
	Instrumental variable: quarter-hour dummy variables			
	Nov. 19, 2009, to Oct. 4, 2010		Oct. 5, 2010, to Aug. 6, 2011	
	Profit of auction	Profit per dollar of product value	Profit of auction	Profit per dollar of product value
Number of actual bidders	8.270***	0.059***	8.644***	0.008***
	(1.912)	(0.014)	(1.675)	(0.003)
Constant	-114.104**	-0.768**	-153.509***	-0.194**
	(45.470)	(0.332)	(47.994)	(0.092)
Observations	22,430	22,430	28,109	28,109
Number of products	671	671	1,086	1,086
R-squared	0.547	0.266	0.607	0.141

Notes: The sample covers regular auctions whose price increment is 1 cent. Product fixed effects are included in all regressions. Reported constants are average product fixed effects. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, and * p<0.1.

Table 6 presents the new IV results for auctions whose price increment is 15 cents. The sample period is from November 19, 2009, to October 4, 2010. The new IV estimates of the marginal effect are also consistent with the IV estimates in Table 4.

When calculating auction revenues in the data section, we made the assumption that the value of a nontoken product without an Amazon price is 78 percent of the retail price posted by

BigDeal. Even though our identification strategy relies on within-product variations, it is still useful to check if our estimates are sensitive to this assumption. We conduct two robustness checks here.

Table 6. Robustness Checks (15-Cent Increment): Alternative IVs

	Instrumental variable			
	Half-hour dummies		Quarter-hour dummies	
	Profit of auction	Profit per dollar of product value	Profit of auction	Profit per dollar of product value
Number of actual bidders	10.957***	0.056***	11.842***	0.057***
	(2.124)	(0.008)	(2.024)	(0.007)
Constant	-120.505***	-0.751***	-132.920***	-0.766***
	(29.806)	(0.108)	(28.403)	(0.104)
Observations	13,792	13,792	13,792	13,792
Number of products	462	462	462	462
R-squared	0.323	0.238	0.338	0.236

Notes: The sample covers regular auctions whose price increment is 15 cents and the period November 19, 2009, through October 4, 2010. Product fixed effects are included in all regressions. Reported constants are average product fixed effects. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, and * p<0.1.

First, we restrict the sample to those products with an Amazon price. Because about 62 percent of the nontoken products have an Amazon price, the restricted sample is still quite large. The new estimates, reported in the "Amazon products only" parts of Tables 7 and 8, remain qualitatively similar to previous estimates.

Second, we use the retail prices posted by BigDeal to approximate product values, and we assume that no bidder ever exercised the BIN option. Recall that we had to estimate which bidders exercised the BIN option when calculating auction revenue. The assumption that the BIN option was never exercised is an extreme way of checking whether our results are sensitive to the way we estimated which bidders exercised the BIN option. The results, reported in the "posted price as value" parts of Tables 7 and 8, are very similar to previous results.

Table 7. Further Robustness Checks (1-Cent Increment)

	Amazon products only			
	Nov. 19, 2009, to Oct. 4, 2010		Oct. 5, 2010, to Aug. 6, 2011	
	Profit of auction	Profit per dollar of product value	Profit of auction	Profit per dollar of product value
Number of actual bidders	11.072**	0.016***	9.360***	0.003
	(4.573)	(0.006)	(1.571)	(0.002)
Constant	-184.878	-0.463***	-166.065***	-0.131**
	(119.792)	(0.158)	(50.733)	(0.054)
Observations	9,032	9,032	10,214	10,214
Number of products	315	315	381	381
R-squared	0.614	0.530	0.601	0.122
	Posted price as value			
	Nov. 19, 2009, to Oct. 4, 2010		Oct. 5, 2010, to Aug. 6, 2011	
	Profit of auction	Profit per dollar of product value	Profit of auction	Profit per dollar of product value
Number of actual bidders	8.644***	0.072***	9.545***	0.051***
	(1.948)	(0.012)	(3.335)	(0.014)
Constant	-105.487**	-0.894***	-127.595	-0.414
	(46.332)	(0.292)	(95.589)	(0.396)
Observations	22,430	22,430	28,109	28,109
Number of products	671	671	1,086	1,086
R-squared	0.514	0.365	0.635	0.389

Notes: The sample covers regular auctions whose price increment is 1 cent. Product fixed effects are included in all regressions. Reported constants are average product fixed effects. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, and * p<0.1.

Table 8. Further Robustness Checks (15-Cent Increment)

	IV estimates			
	Amazon products only		Posted price as value	
	Profit of auction	Profit per dollar of product value	Profit of auction	Profit per dollar of product value
Number of actual bidders	8.998***	0.056***	11.372***	0.073***
	(2.427)	(0.008)	(2.770)	(0.009)
Constant	-98.558***	-0.750***	-130.742***	-0.991***
	(30.025)	(0.102)	(38.867)	(0.126)
Observations	9,128	9,128	13,792	13,792
Number of products	296	296	462	462
R-squared	0.287	0.239	0.216	0.259

Notes: The sample covers regular auctions whose price increment is 15 cents and the period November 19, 2009, through October 4, 2010. Product fixed effects are included in all regressions. Reported constants are average product fixed effects. Robust standard errors are in parentheses. *** p<0.01, ** p<0.05, and * p<0.1.

6. Conclusion

How does competition affect the revenue in all-pay contests? This paper takes the first step toward documenting empirical evidence on this relationship from the field. We study penny auctions, a rare form of all-pay contest that generates enough data to make it possible to empirically study this question. In particular, the setting that we study allows us to use novel instruments to overcome the critical econometric issue that the number of observed bidders in an auction is endogenous.

We find strong evidence that increased competition increases the revenue of a penny auction. Our findings are inconsistent with models of penny auctions that presume that all bidders are fully informed and rational. Our findings could result from bidders' incorrect beliefs about the number of competitors, from their inability always to act optimally, or from their private valuations of the products sold. It would be interesting for future research to build and test more general models of penny auctions that can explain our findings.

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